1. **Introduction:** The problem “Building a Stadium” is as follows:

*In the state of North Carolina, a specific region in the Piedmont area is known as the “Research Triangle” because its general geographic region is contained between three cities: Raleigh (approximate population: 380,173), Durham (approximate population: 479,624), and Chapel Hill (approximate population: 54,492). Suppose the state is planning on bringing in a professional baseball team and needs to build a new baseball stadium. An aerial map of the area is shown below with a second more simplified map of the area that can me used for the purpose of working this problem. Imagine you are on the board that will decide on where the stadium will be located. The contractor has determined the cost of building new highways is $125,000 per mile and the cost of resurfacing an existing highway is $50,000 per mile. In the simplified map, let centimeters represent miles.*

In this map, centimeters are being used to represent miles, point R is being used to represent Raleigh, point D is being used to represent Durham, and point C is being used to represent Chapel Hill. As this simplified map above shows, the distance between Raleigh and Durham is 28.5 miles. The distance between Durham and Chapel Hill is 11.1 miles, and the distance between Chapel Hill and Raleigh is 32.2 mile. RD represents Highway US-70, CD represents Highway US-15-501, and RC represents Interstate I-40.

To solve this problem, consider different centers of the triangle (centroid, circumcenter, incenter, and orthocenter) that might be used as the location of the stadium. The centroid is the point where the three medians meet. The circumcenter is the point where the perpendicular bisectors of the sides meet. The incenter is the point where the angle bisectors meet. The orthocenter is the point where the altitudes of the triangle meet. Which center of the triangle would be the most economical to use for the location of the new baseball stadium?
2. **Student Solution:**
First, I investigated the centroid of the triangle, using The Geometer’s Sketchpad, shown in the figure below as point E. I used a screen capture tool to copy and then insert each of the images of the triangle. I found the centroid by connecting the midpoint of each side to its opposite vertex and finding the intersection, E, between these three lines.

I then decided to calculate other distances between the cities and the centroid, shown as segments ER, ED, and CE above. If the new baseball stadium were to be built at the location of the centroid, the shortest path between each city and the stadium would be these segments. Since highways do not already exist along each of these segments, new highways would have to be built. This would cost $125,000(19.93 + 9.63 + 12.94) = $5,312,500.

To minimize cost, subsequently increasing travel time for baseball fans, the existing highways could be used to get to the midpoint of the highway segment, and then a smaller amount of new highway could be built from the midpoint to the centroid, where the stadium would be located. I calculated these distances, shown below as segments AR, AE, EF, CF, GE, and GD.
If the new highways were built along AE and GE, it would cost $125,000(6.47 + 9.97) = $2,055,000. Then the existing highways would have to be repaved. The shortest route to the stadium from Durham would be along the segment of Highway US-15-501 labeled GD, then on the new highway GE. The shortest route from Chapel Hill would be along the other segment of Highway US-15-501 labeled CG, then again along GE. The shortest route from Raleigh would be along the segment of Highway US-70 labeled AR, then along the new highway AE. The cost to repave these portions of highway would be $50,000(14.25 + 11.1) = $1,267,500. Thus, the total cost would be $2,055,000 + $1,267,500 = $3,322,500. This total cost is less than the cost of only building new highways, so this method would be more economical than building new highways directly between each city and the stadium, if located at the centroid of the triangle.

Next, I will investigate the circumcenter of the triangle, again using The Geometer’s Sketchpad. I again used the midpoints of each side, but this time I created perpendicular lines from each midpoint, and the intersection between these three lines is the circumcenter, as shown below in point I.

If the point I were used as the location of the new stadium, the portion of Interstate I-40 labeled RH would have to be repaved, and a highway would have to be built at the segment HI. The portion of Interstate I-40 labeled CJ would also have to be repaved, and a highway would need to be built at the segment JI. The segment of Highway US-15-501 CD would also have to be repaved. The cost of this would be $50,000(15.15 + 11.41 + 11.1) + $125,000(2.78 + 5.37) = $2,901,750 for the repaving and building of these highways in order for the new baseball stadium to be located at the circumcenter of the triangle. The distance between each vertex and point I is 16.31 miles, so the cost to build new highways between each city and the circumcenter would be $125,000(3(16.31)) = $6,116,250. Again, using the existing is a more economical method than only building new highways between each city and the location of the stadium.
Next, I will investigate the incenter of the triangle, again using The Geometer’s Sketchpad. I found the incenter by finding the intersection of the three lines that bisect each angle of the triangle, shown below as point E.

Again, the fastest route to the stadium, if built at the incenter, would be along new highways directly from each city, labeled as the segments ER, DE, and CE on the image above. The cost to build these three new highways would be $125,000(25.18 + 8.58 + 5.71) = $4,933,750. Next, I will see if using the existing highways will be more cost efficient, as it was for the centroid.
Now, the segment of Highway US-70 labeled RF would be repaved, as would CD of Highway US-50-501. Then new highways would need to be built along FE and GE. This would lead to a cost of $50,000(21.19 + 11.1) + $125,000(5.65 + 4.6) = $2,895,750 to repave and build the highways for the new stadium to be located at the incenter of the triangle. As with the centroid and circumcenter, this method is more cost efficient than only building new highways.

Finally, I will investigate the orthocenter of the triangle, also using The Geometer’s Sketchpad. The orthocenter is the point where the three altitudes of the triangle intersect, shown below as point E.

Again there are two options to consider if this point where used to build the new baseball stadium. First, new highways can be built from each city to the stadium, along segments ER, ED, and CE. This would result in a cost of $125,000(30.67 + 5.23 + 15.87) = $6,471,250 which is a higher cost than any result already found. The other option is to use the existing Highways US-70 and US-15-501 and only build a small new highway from Durham, point D, to the new stadium. This would cost $50,000(28.5 + 11.1) + $125,000(5.23) = $2,633,750. This appears to be the most cost efficient, but now I want to organize my results into a table to better compare them.

<table>
<thead>
<tr>
<th>Triangle Centers</th>
<th>Creating all new highways</th>
<th>Utilizing existing highways</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centroid</td>
<td>$5,312,500</td>
<td>$3,322,500</td>
</tr>
<tr>
<td>Circumcenter</td>
<td>$6,116,250</td>
<td>$2,901,750</td>
</tr>
<tr>
<td>Incenter</td>
<td>$4,933,750</td>
<td>$2,895,750</td>
</tr>
<tr>
<td>Orthocenter</td>
<td>$6,471,250</td>
<td>$2,633,750</td>
</tr>
</tbody>
</table>
As seen in the table above, constructing the new baseball stadium at the orthocenter of the triangle will be the most economical option, with only 39.6 miles of highway repaved and 5.23 miles of new highway built. This results in the stadium located at approximately the blue point, the orthocenter, on the map below. The pink lines outline the triangle, and the purple lines represents the altitudes of the triangle. The green dashed lines represent the highways that would need to be repaved or built for the construction of the new stadium. To create this map, I took a screen capture of the map and drew the lines in a picture editing application.
3. **Teacher Discussion:**
The “Building a Stadium” problem addresses standards from the High School Geometry course in the Alabama College and Career-Ready Standards. This problem addresses the mathematical topics of congruence and modeling with geometry. The topic entitled ‘make geometric connections’ under the congruence topic includes the standard [G-CO12], which is greatly used in this problem. The standard is as follows: “Make formal geometric constructions with a variety of tools and methods such as…dynamic geometric software.” The standard also gives examples of such constructions, which includes bisecting an angle, constructing perpendicular lines, and constructing the perpendicular bisector of a line segment. Each of these constructions was done in this problem using the dynamic geometric software of The Geometer’s Sketchpad. This problem also addresses the topic of applying geometric concepts in modeling situations, listed under the modeling with geometry topic. The first standard in this category states: “Use geometric shapes, their measures, and their properties to describe objects.” [G-MG1]. In this problem, we are using a triangle and its side measures to describe the three cities and the distances via highways between them. Then the properties of the centers of the triangle are used to come to a conclusion for the problem. Another standard in modeling with geometry states: “Apply geometric methods to solve design problems,” [G-MG3]. Geometric methods involving the centers of triangles were used to solve the building a stadium problem.

In the classroom, I think this activity would be a good project for Geometry students. It would be best to assign this project after discussing triangles and teaching about the different centers. This project would allow students to show that they understand how to construct each center of the triangle. It would also allow students to use dynamic geometric software to investigate and model the situation. Since Geometer’s Sketchpad is not free, the students may need to be advised to use a free application such as Geogebra to perform their investigation. Alternatively, this project could be done partially in the classroom or computer lab so that each student has equal access to the same technology. Then, the students could finish the write up of their results and come to their final conclusion for an at-home portion of the project. I enjoyed working through this problem since it was an interesting connection to the real world. I believe that high school students in Geometry class would also enjoy this project, especially when considering how alternative geometry projects often lack such interesting connections. This project would be beneficial to students because it gives them the opportunity to use dynamic geometric software as well as investigate an important aspect of geometry in a fun way.

In *Focus in High School Mathematics: Technology to Support Reasoning and Sense Making*, we read about conveyance and mathematical action technologies. For the “Building a Stadium” problem, the use of the mathematical action technology The Geometer’s Sketchpad was an integral part of developing a solution. The use of this technology enhanced the understanding of mathematical connections much more than if mathematical action technology had not been used. In the article *Engaging Geometry Students through Technology*, dynamic geometry software, such as The Geometer’s Sketchpad used in this problem, were discussed as an appropriate and helpful tool for students. The article mentions using dynamic geometry software to approach seemingly unreachable tasks. The task for building a stadium may be considered an unreachable task without the assistance of such software.
References

